Mathematics is All Around Us

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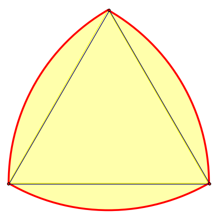
Why are sewer covers round?

Because they can then never fall in the hole.

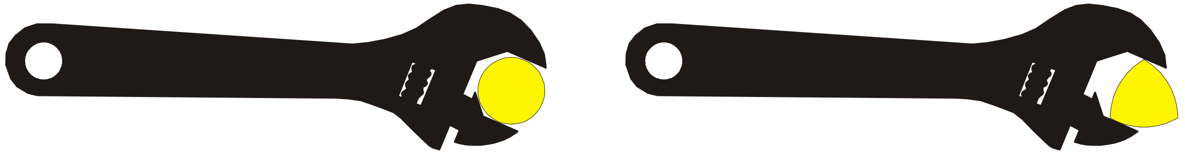
Which other geometric shape can also be used?

The German engineer **Franz Reuleaux** (1829 – 1905) developed a shape that has properties similar to a circle.

The shape that is now called a ***Reuleaux triangle***.



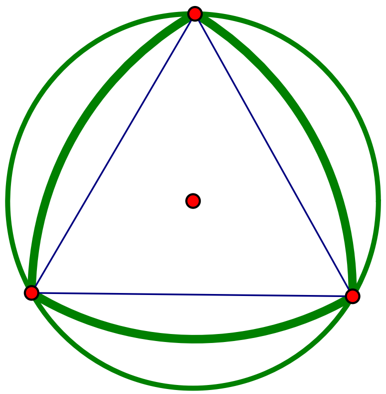
All opposite points are everywhere equidistant.



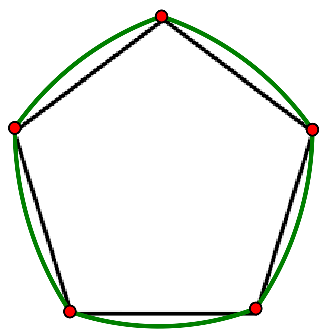
Therefore, it can be used on a fire hydrant.

- …and it can also be used as a sewer cover!

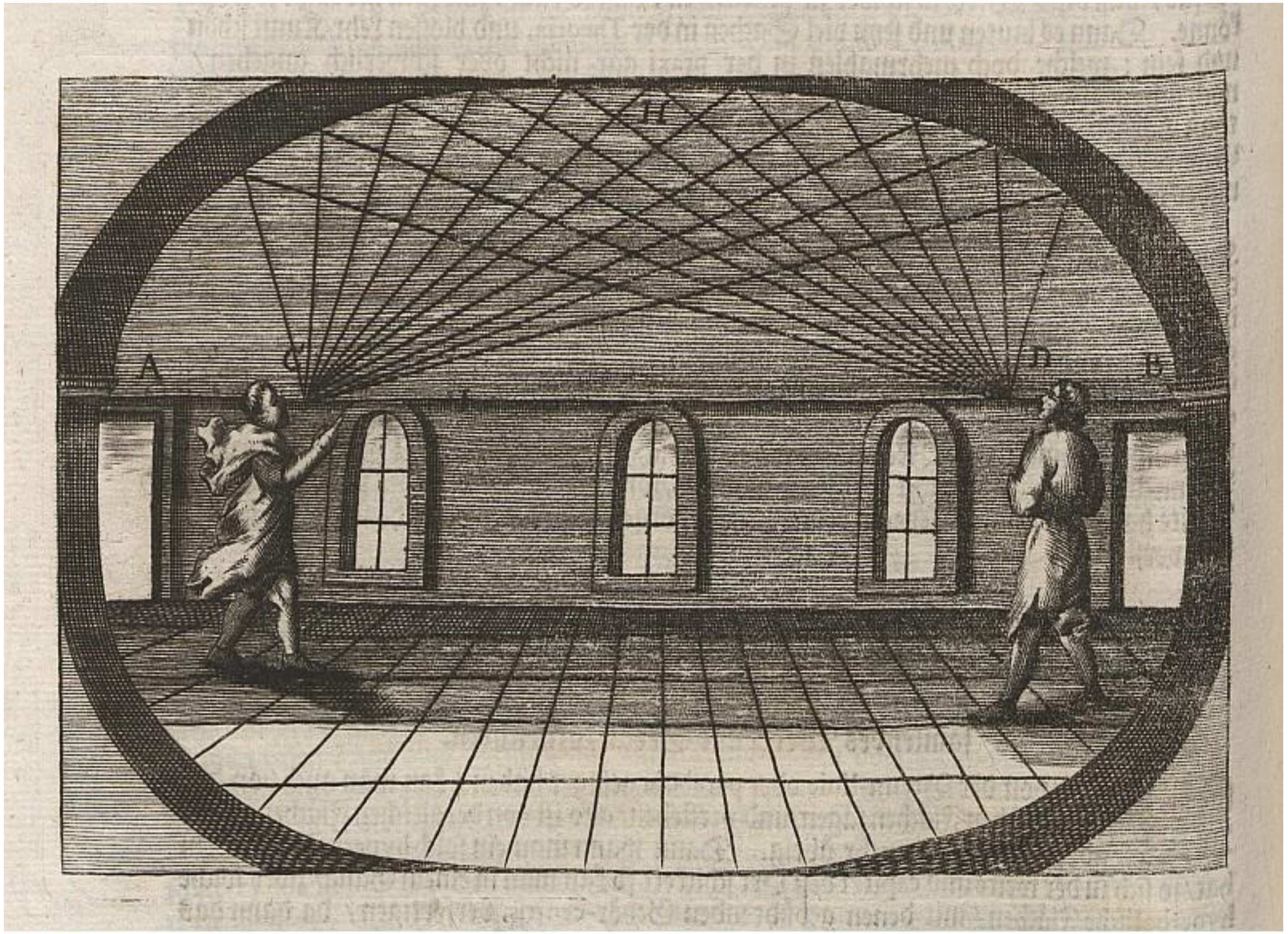
The Austrian mathematician Wilhelm Blaschke (1885-1962) proved that given any number of such figures of equal breadth, the **Reuleaux triangle** will always possess the *smallest area*, and the **circle** will have the *greatest area*.



Here is another figure of constant breadth.



Whispering Points

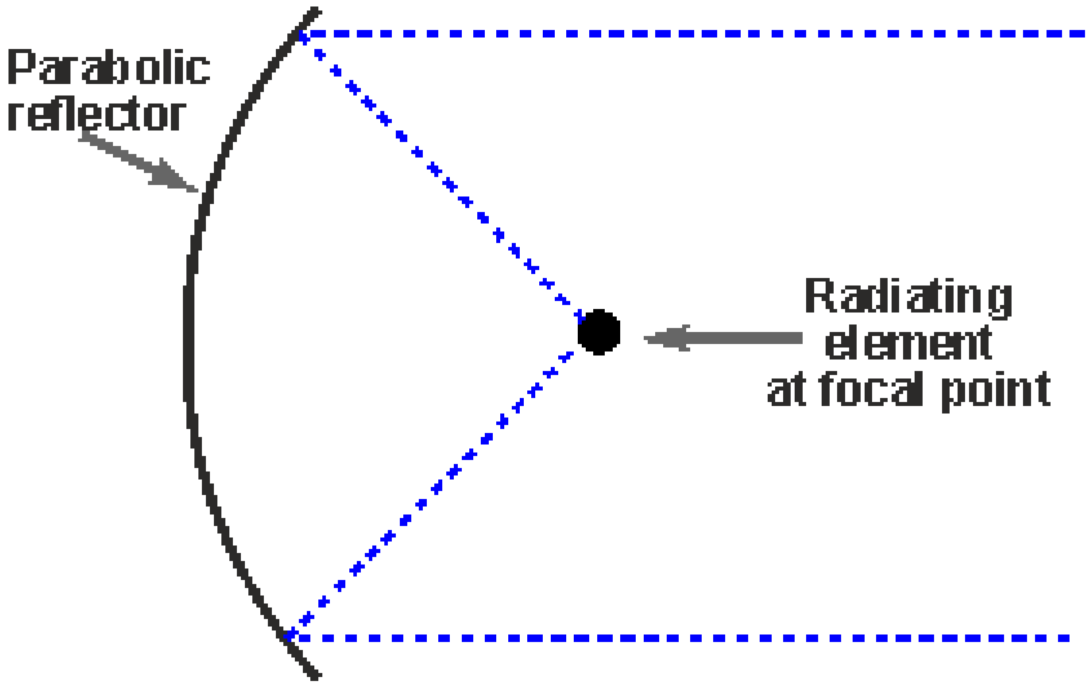


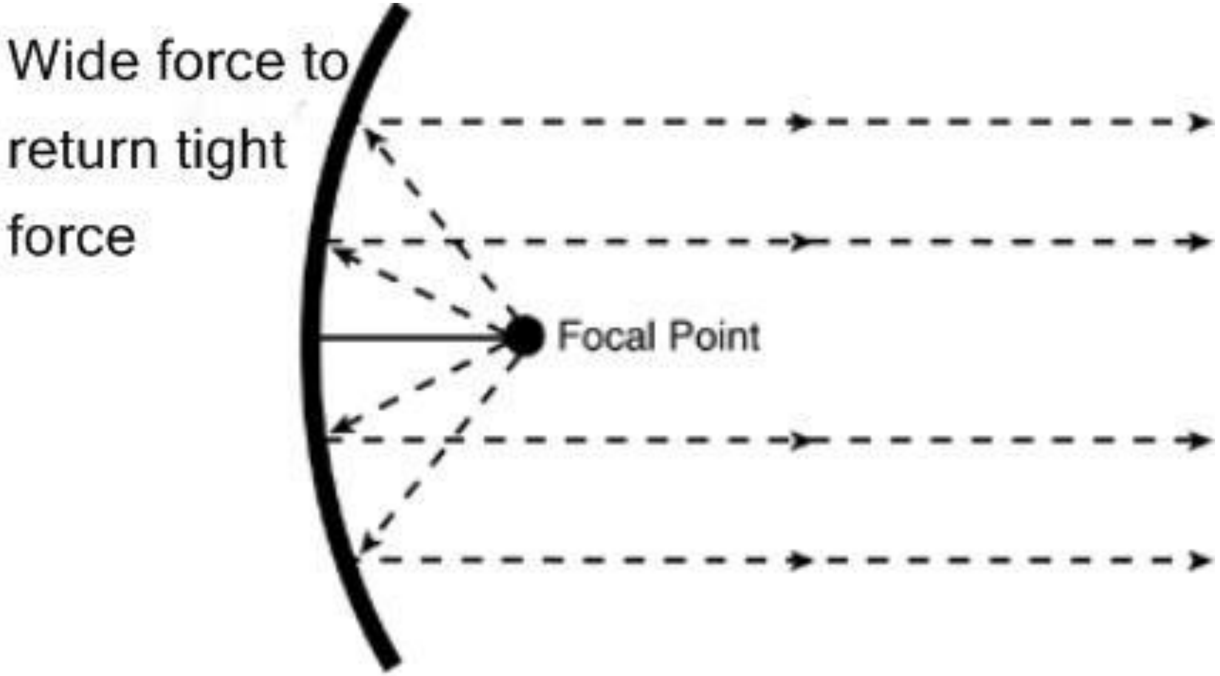
Quelle: Deutsche Fotothek

Whispering spots
at Grand Central Railroad station, New York

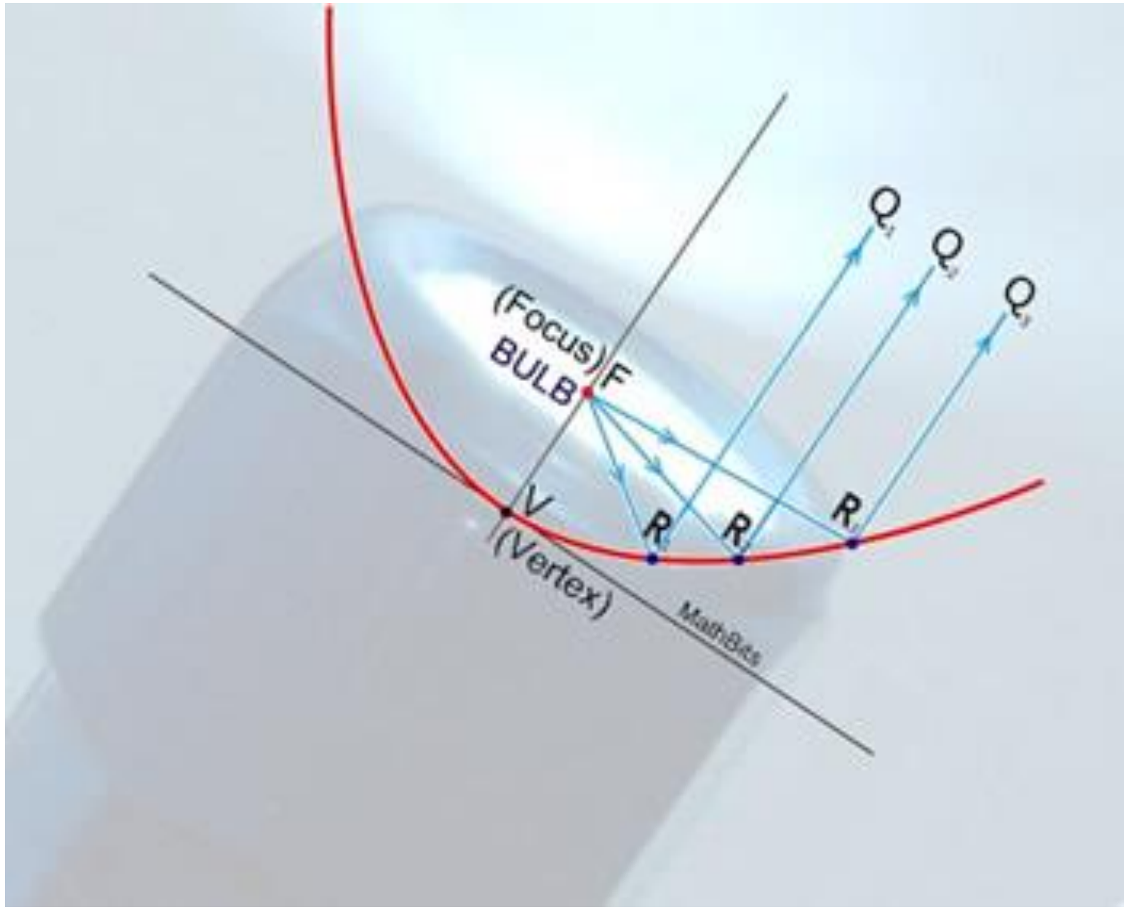
Whispering spots in the Statuary Hall of the Capitol Building, Washington, D.C.

Parabolic Reflector

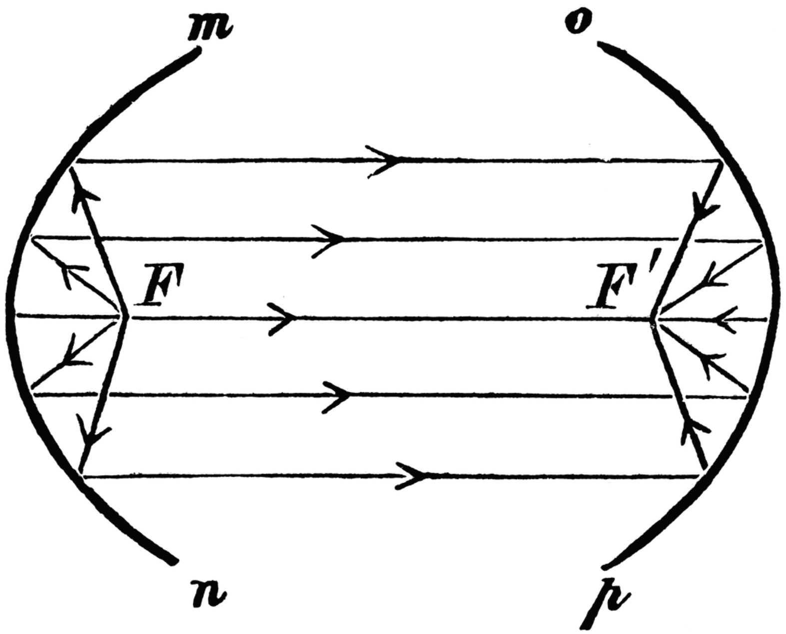




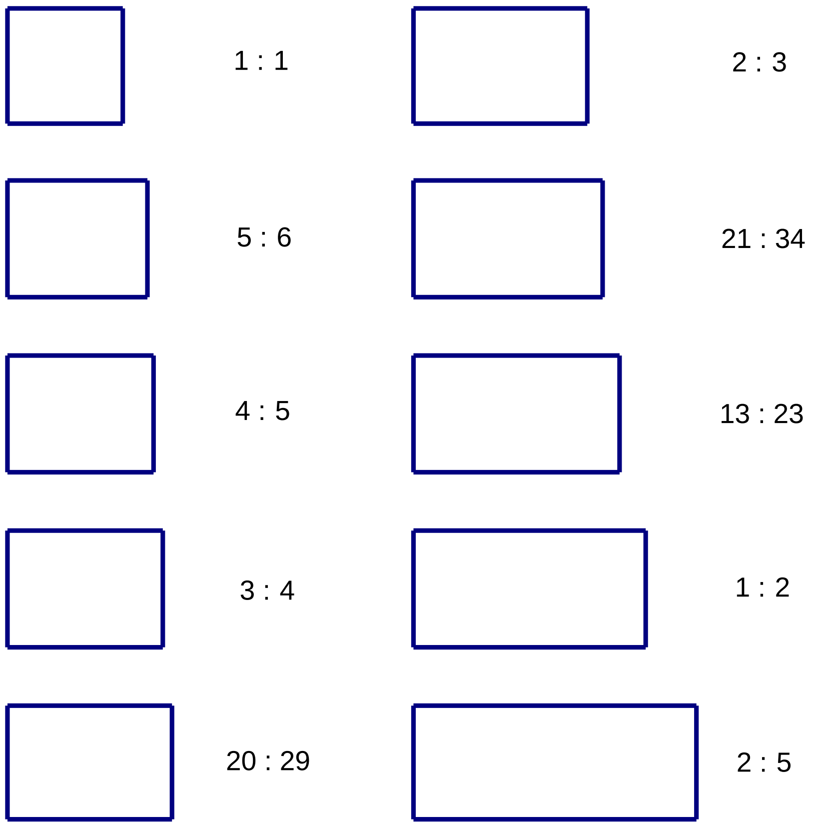
Parabolic reflector of a flashlight



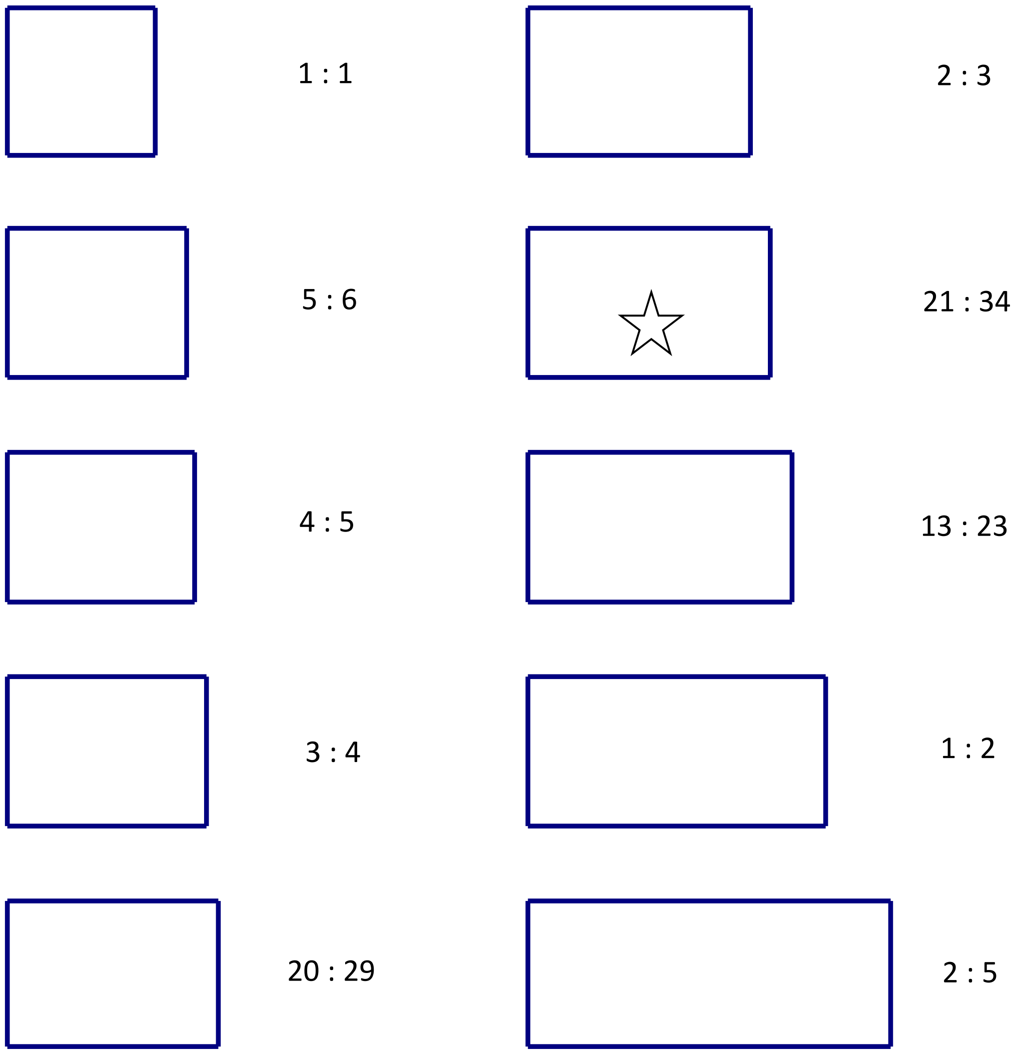
Dual Parabolic Reflector as can be found at a playground



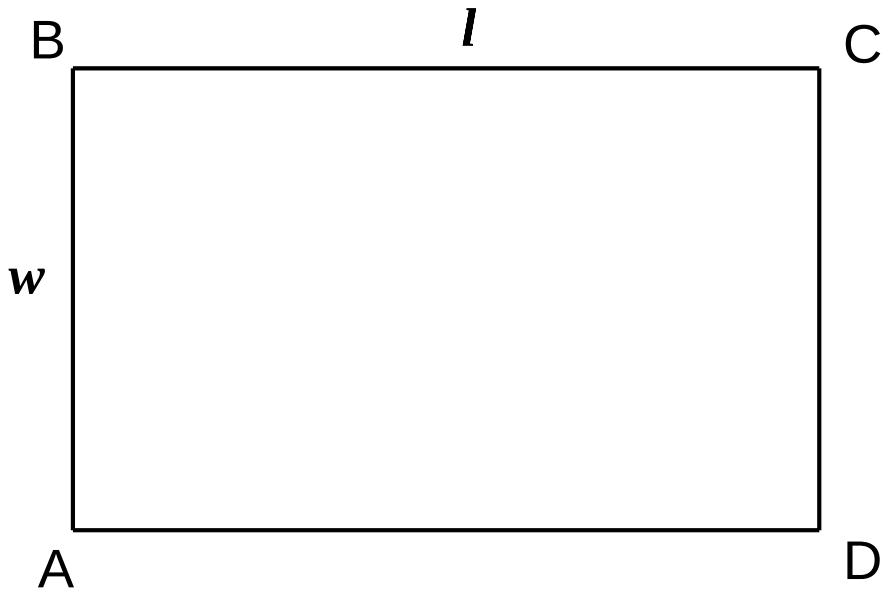
Which rectangle is most pleasing to look at?



Was this your choice?



The Golden Ratio



$$\frac{width}{length} = \frac{length}{width + length}$$

The Golden Ratio

$$\phi = \frac{length}{width} = \frac{Width + length}{length} = \frac{width}{length} + \frac{length}{length}$$

$$\phi = \qquad \frac{1}{\phi} \quad + \quad 1$$

$$\phi^2 - \phi - 1 = 0$$

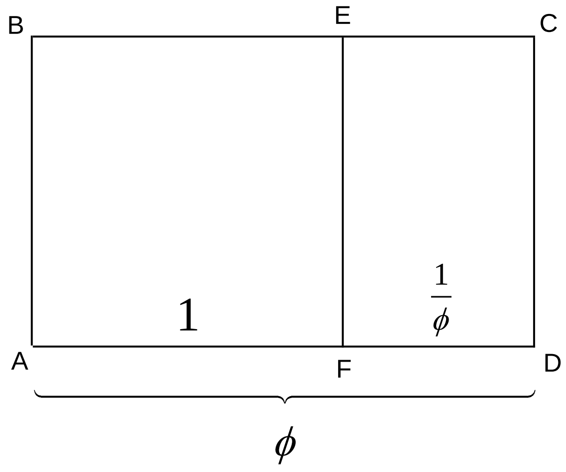
Applying the quadratic formula we get: $\phi = \frac{1 \pm \sqrt{5}}{2}$

$$= \frac{1+\sqrt{5}}{2} = 1.61803398874989484820458683436 56$$

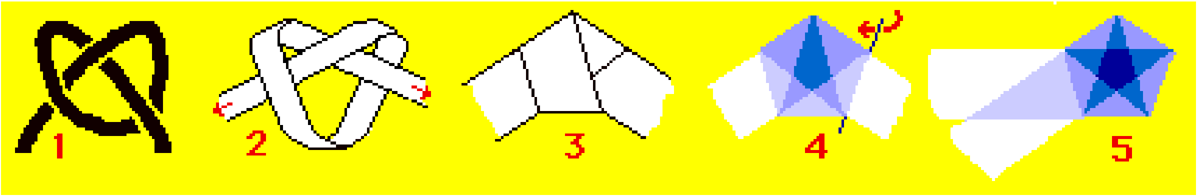
Remember:

The Golden Ratio, ϕ, is the *only* number which is 1 greater than its reciprocal.

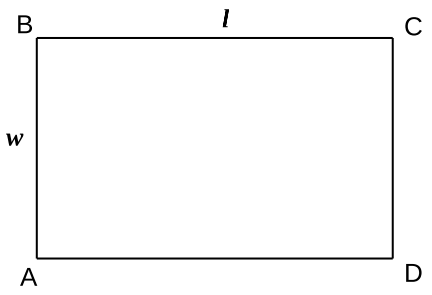
$$\phi = 1 + \frac{1}{\phi}$$

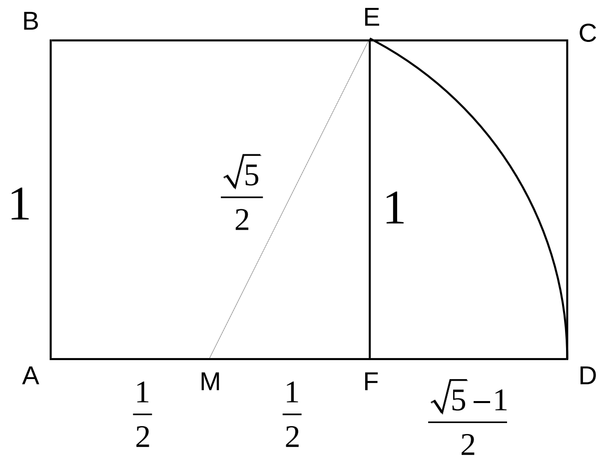


A typical knot leads to the Golden Ratio

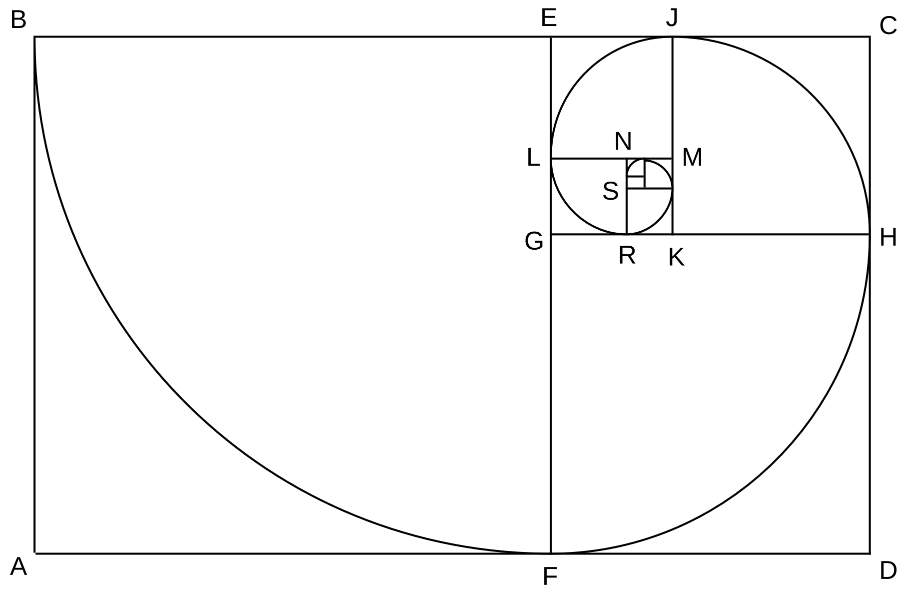


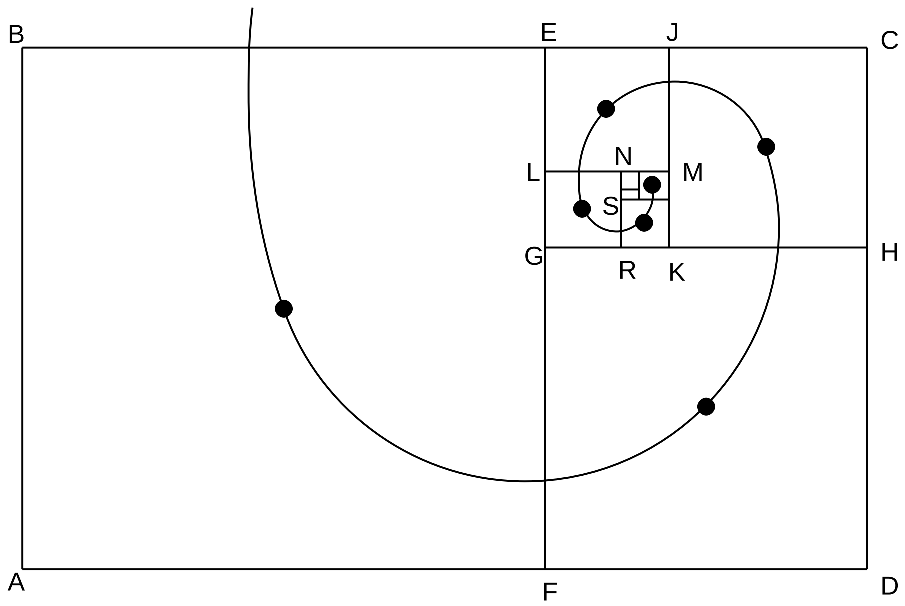
Constructing a Golden Rectangle



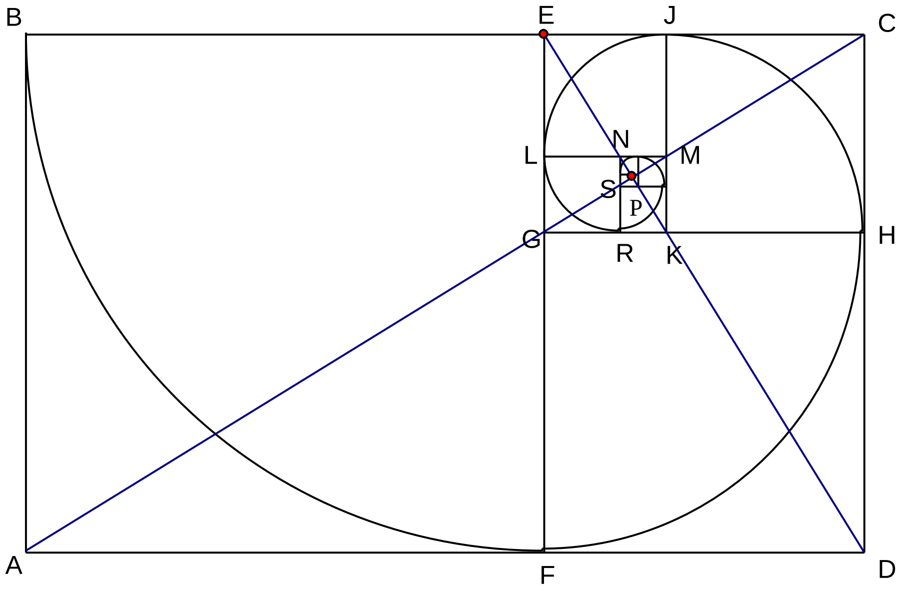


Creating a Golden Spiral



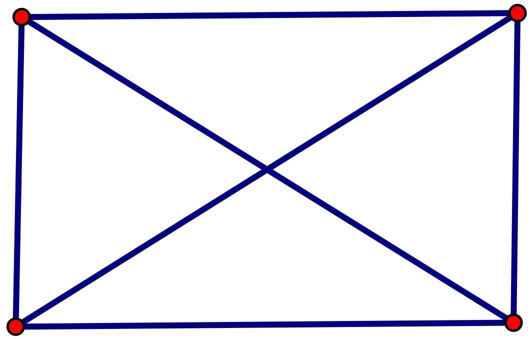


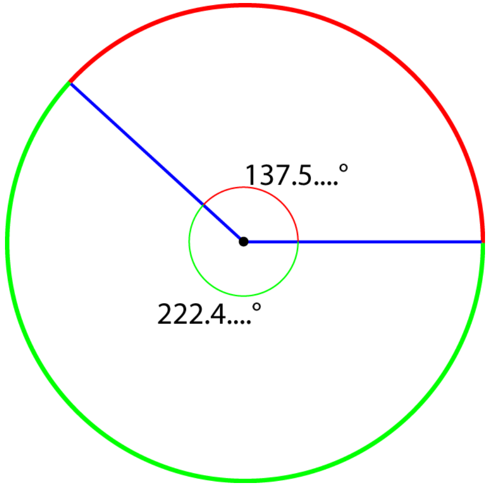
Locating the Vanishing Point



A Spiral in Nature

The Golden Angle

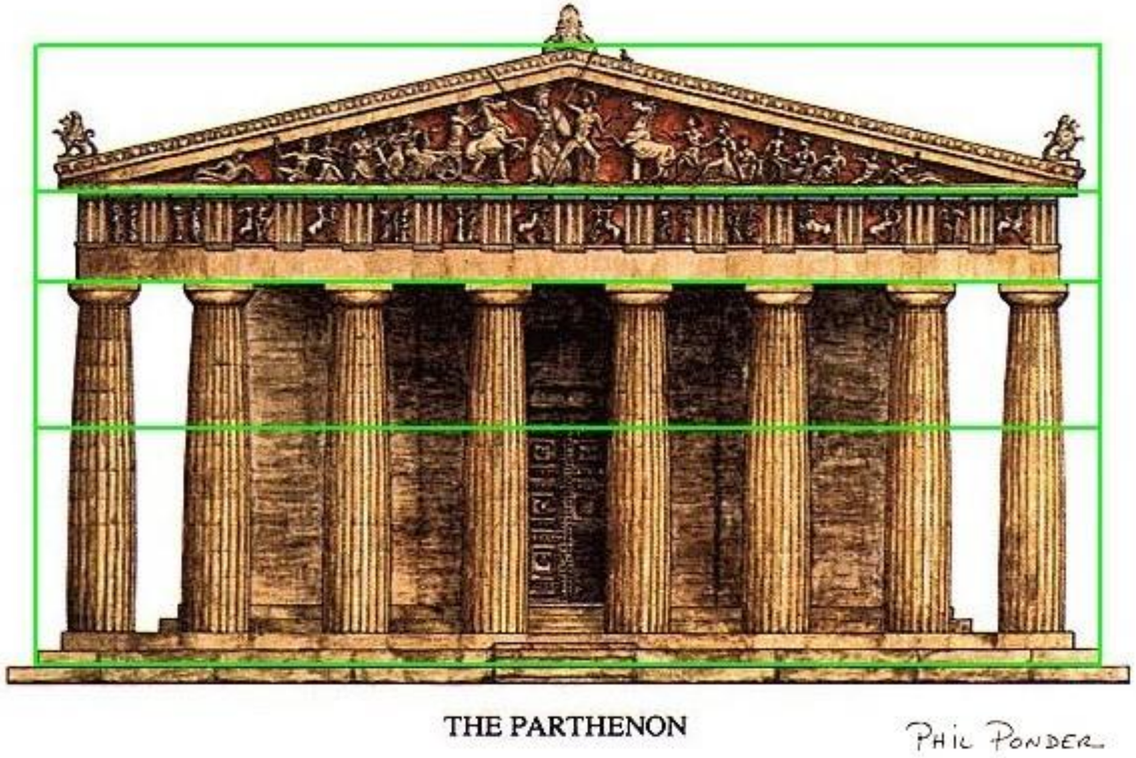




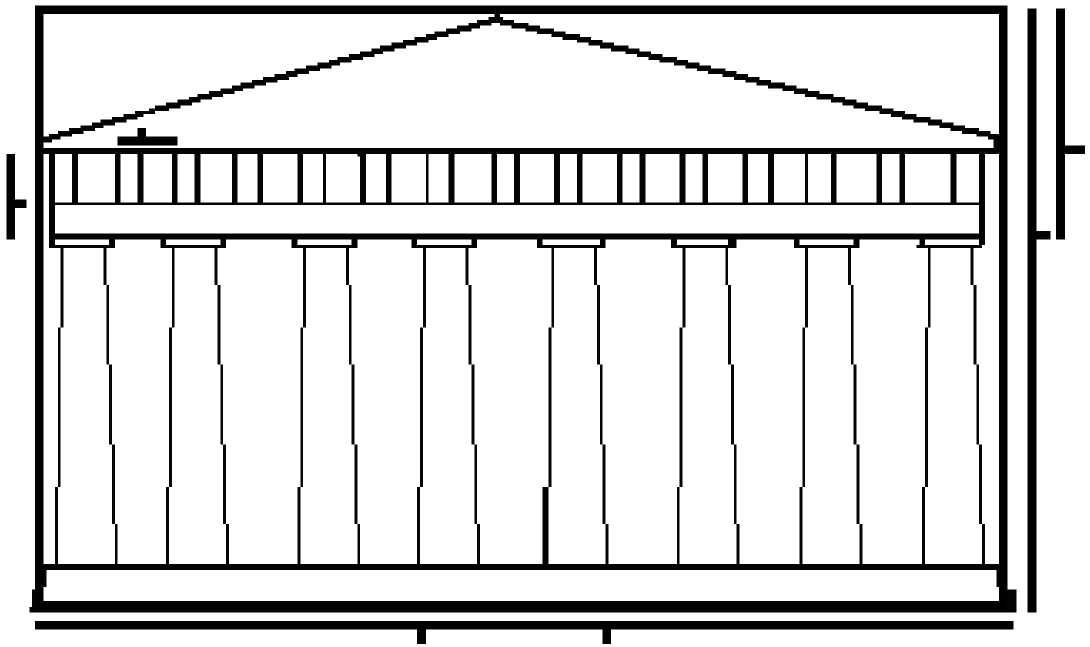
$\frac{360°}{\phi}$ = 222.4...°and 360° – $\frac{360°}{\phi}$ = 360°(2 – ϕ) =137.5...°

Common Clock Pictures – Using the Golden Angle

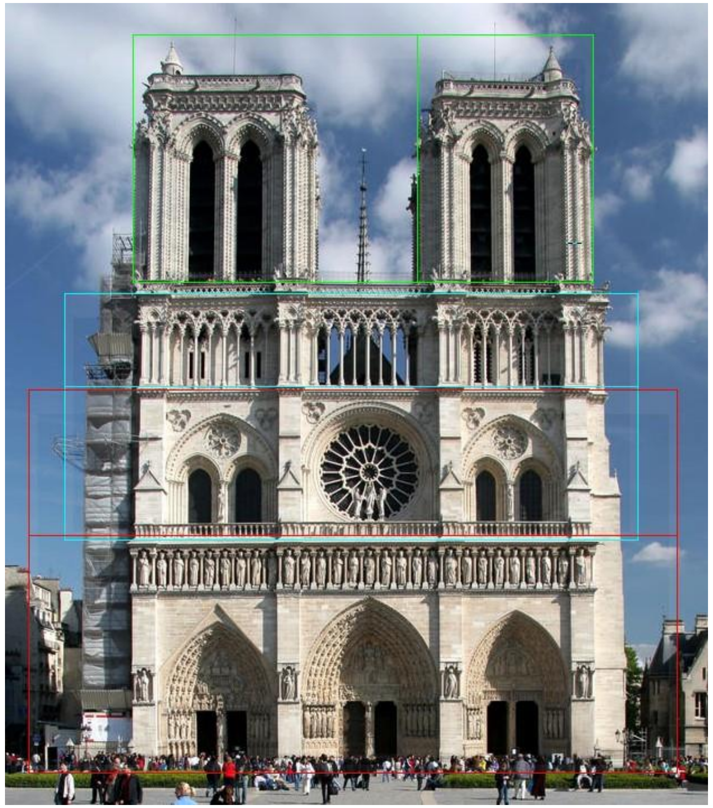
The Golden Rectangle Used in Architecture

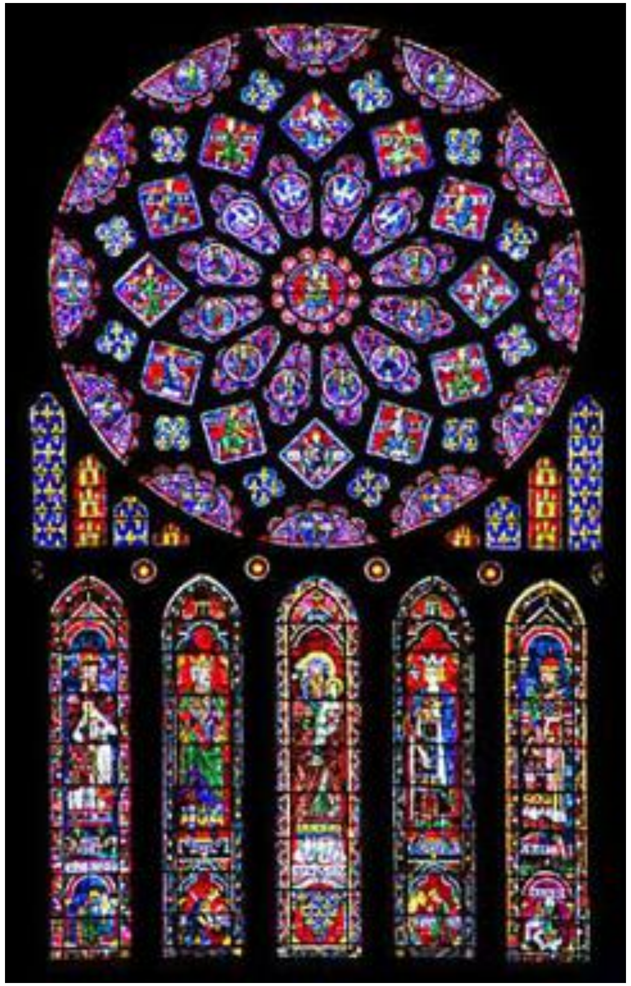


A Sketch of the Golden Rectangle Superimposed on the Parthenon



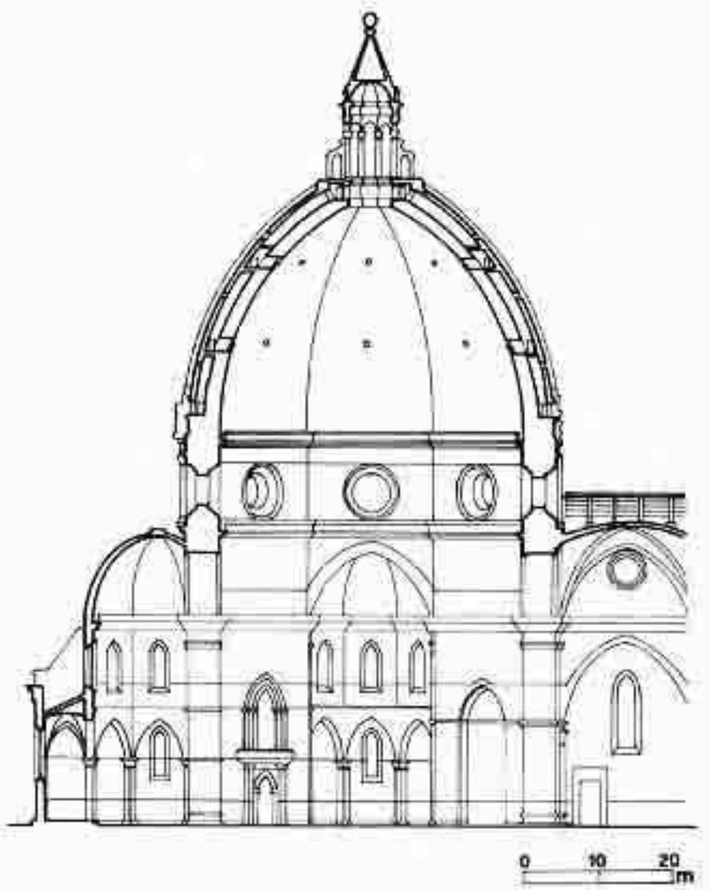
More Architectural Applications of the Golden Rectangle

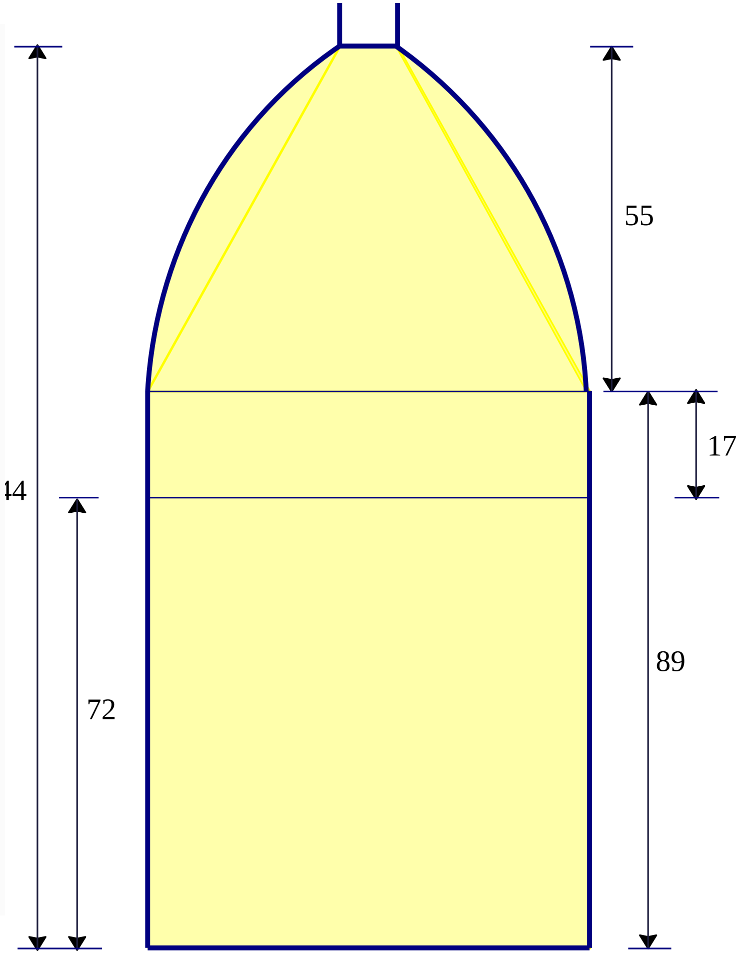




Il Duomo di Firenze exhibits the Fibonacci numbers

Florence Cathedral - View of the dome.

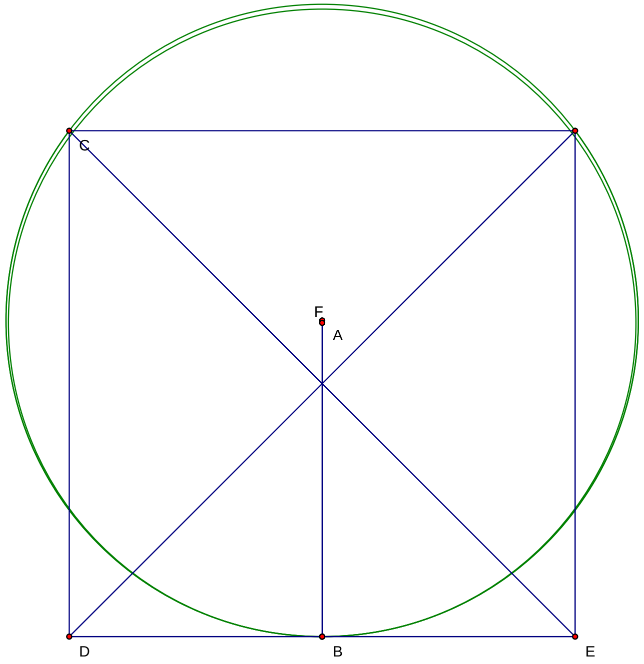


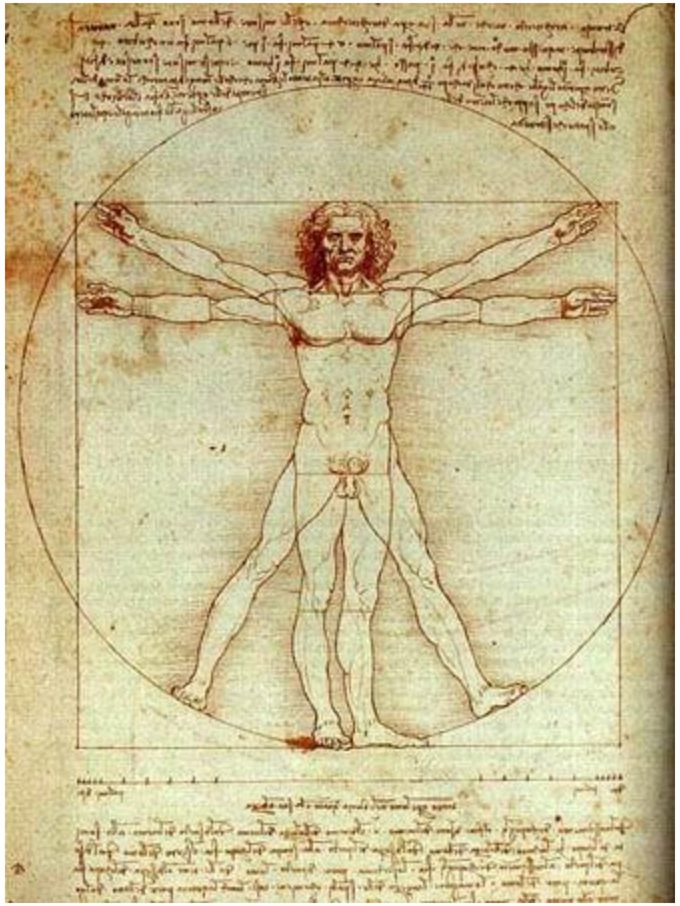


The United Nations building in New York City

Leonardo da Vinci (1452-1519) illustrated the book *De divina proporzione* by Fra Luca Pacioli (ca. 1445-1517) with an anatomical study of the "Vitruvian" man. This drawing supported Pacioli's discussion of the Roman architect Vitruvius (ca. 84 B.C.E. – 27 B.C.E.).

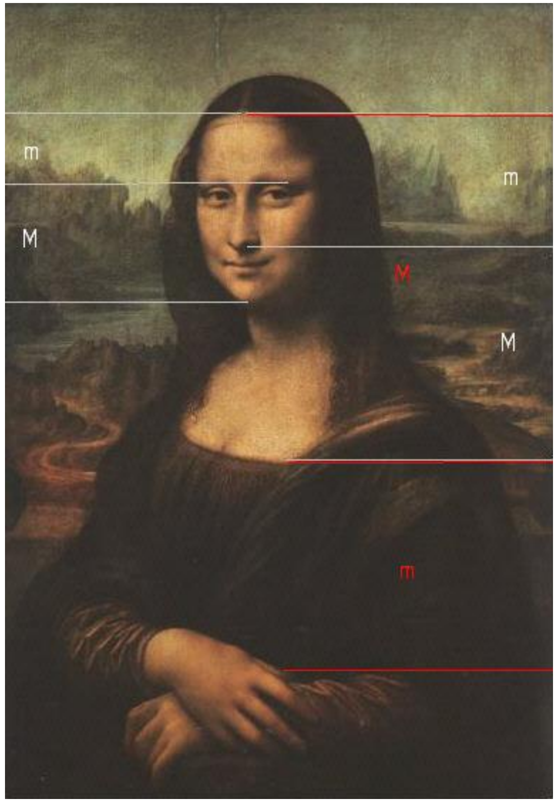
$$\frac{\text{Side of square}}{\text{Radius of circle}} \approx \phi, \text{ which is the Golden Ratio}$$

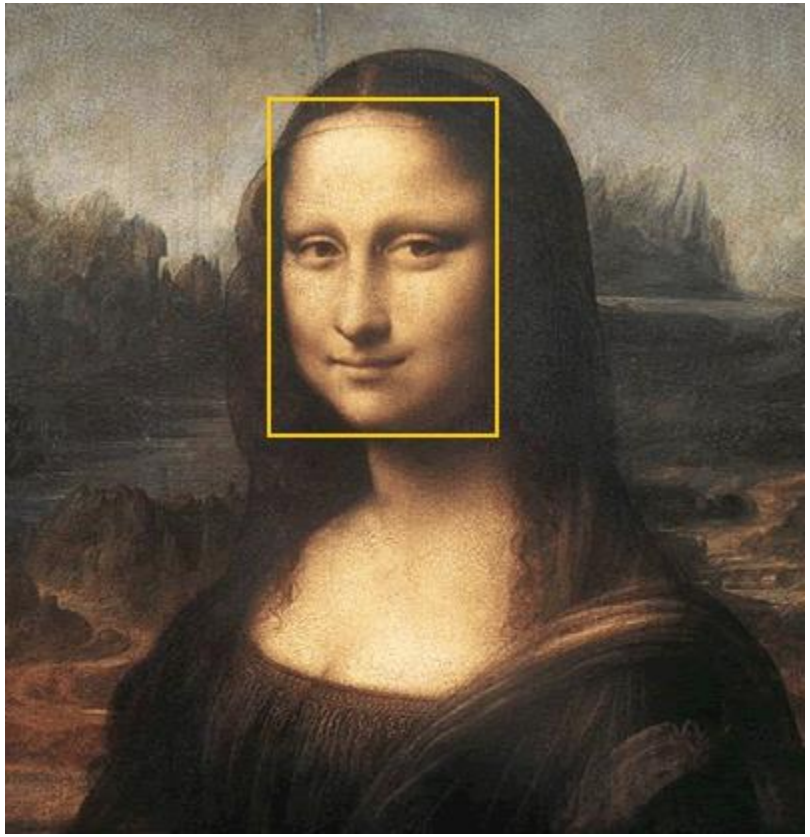




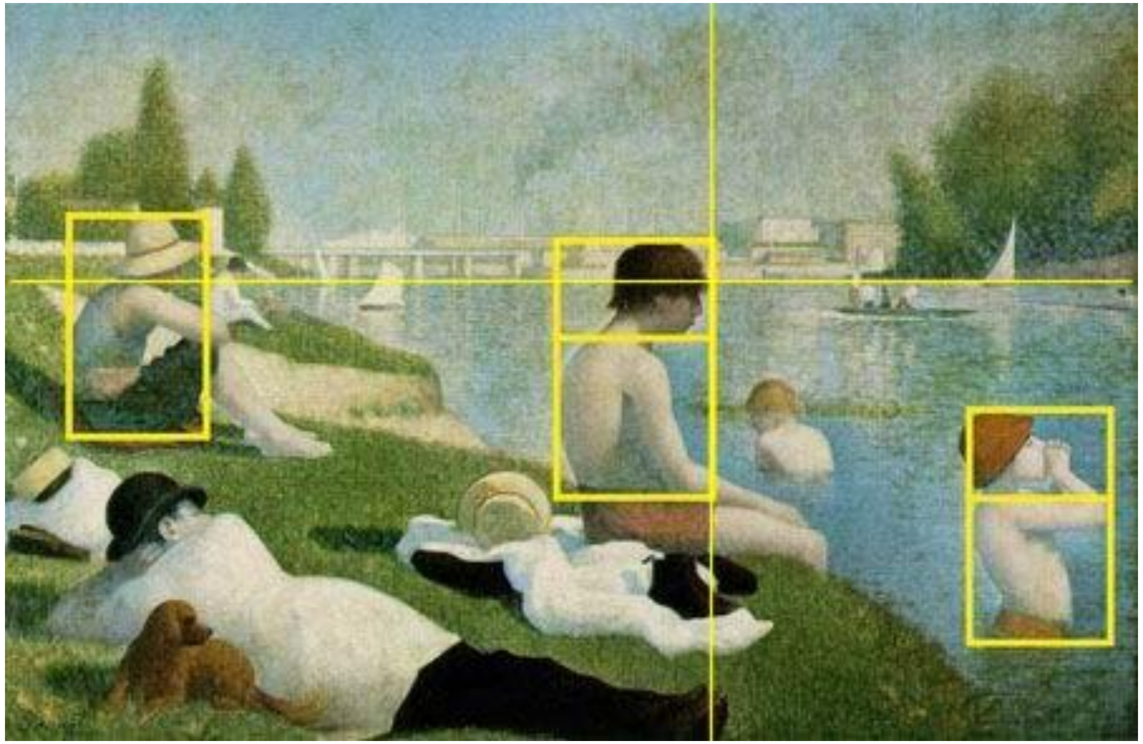
Portrait of Luca Pacioli

Mona Lisa is also proportioned “goldenly”

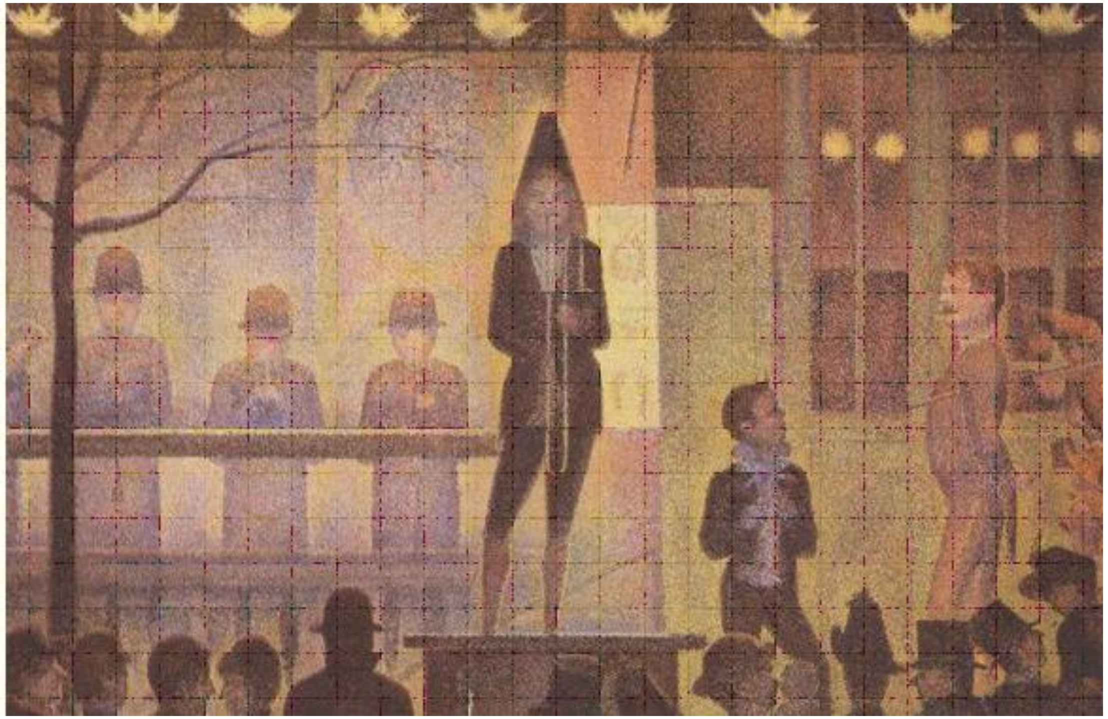




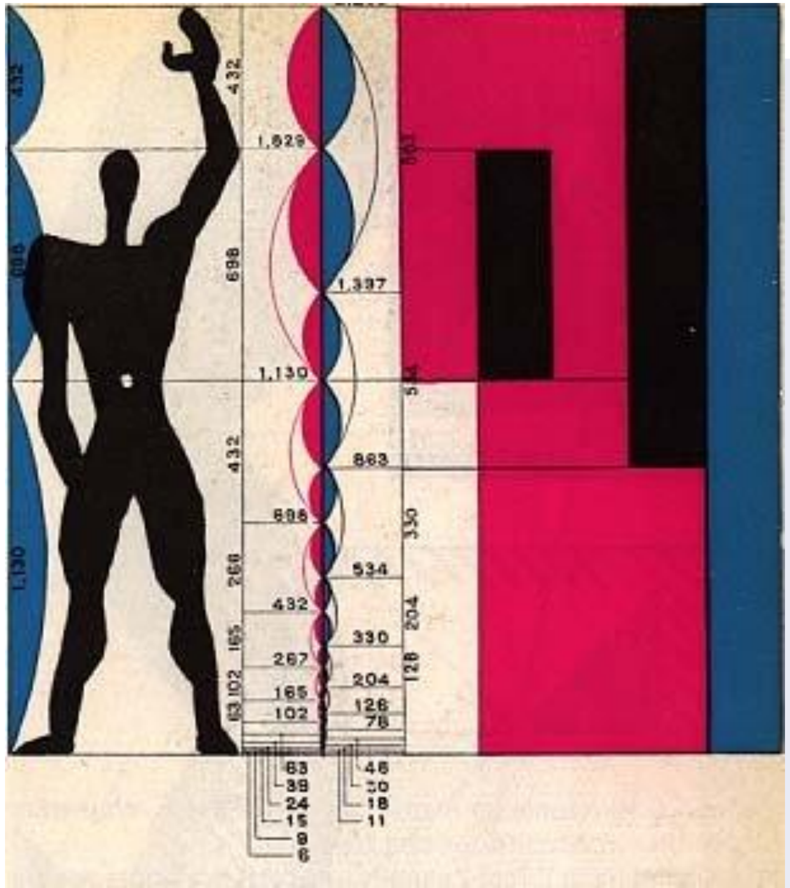
Bathers at Asmieres by Georges Seurat



The *Circus Parade* by Georges Seurat



Modulor by Le Corbusier

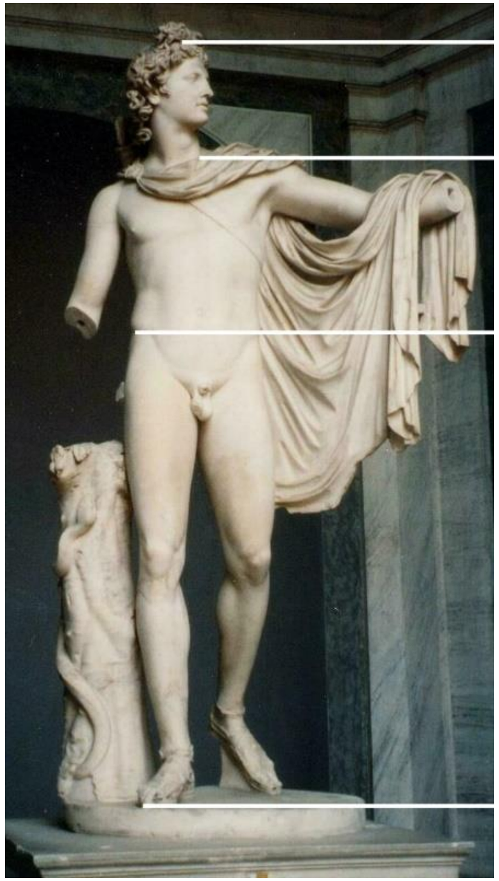


1829	
1130	2260
698	1397
432	863
267	534
165	330
102	204
63	126
39	78
24	48
15	30
9	18
6	11
(*)	(**)

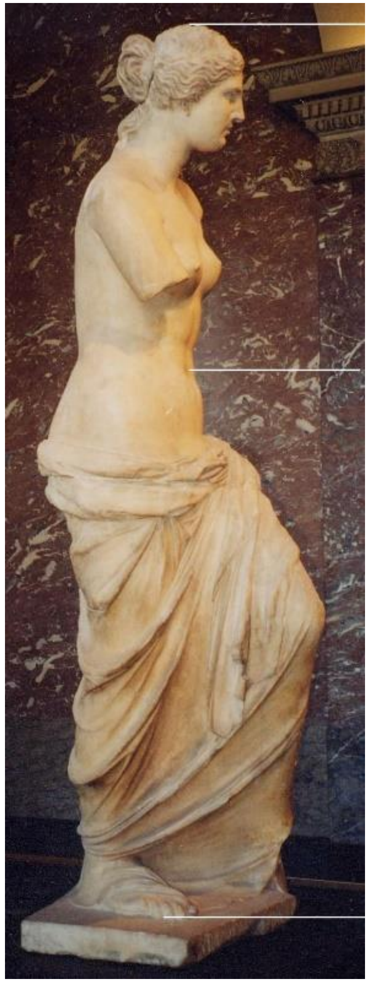
Modulor_1.bmp
Le Corbusier: Modulor

The oldest building in Germany: Königshalle in Lorsch built in 770.

Apollo Belvedere



Venus de Milos



What is the probability that in a room with 35 people there will be 2 with the same birthdate?

Perhaps 2 out of 365?

That would be a probability of $\frac{2}{365} = .005479 \approx \frac{1}{2}\%$

This is a minuscule chance.

Let’s consider a randomly selected group.

From the “randomly” selected group of the first 35 United States presidents

there are **two** with the same birth date**:**

The 11th president, James K. Polk (November 2, 1795),and

The 29th president, Warren G. Harding (November 2, 1865).

Let's consider a group of 35 people.
What do you think is the probability that one selected person matches his own birth date?
Clearly *certainty*, or 1.
This can be written as . $\frac{365}{365}$

The probability that another person does *not* match the first person is . $\frac{365-1}{365}=\frac{364}{365}$

The probability that a third person does *not* match the first and second person is . $\frac{365-2}{365}=\frac{363}{365}$

The probability of all 35 people *not* having the same birth date is the product of these probabilities:

$$p=\frac{365}{365}\bullet\frac{365-1}{365}\bullet\frac{365-2}{365}\bullet\cdots\bullet\frac{365-34}{365}.$$

Since the probability (q) that two people in the group <u>have</u> the same birth date and the probability (p) that two people in the group do <u>not</u> have the same birth date is a certainty, the sum of those probabilities must be 1.
Therefore, $p + q = 1$.

In this case, $q=1-\frac{365}{365}\bullet\frac{365-1}{365}\bullet\frac{365-2}{365}\bullet\cdots\bullet\frac{365-33}{365}\bullet\frac{365-34}{365}\approx .8143832388747152$

Which is about 8 out of 10 times! WOW!!!

Number of people in group	Probability of a birth date match
10	.1169481777110776
15	.2529013197636863
20	.4114383835805799
25	.5686997039694639
30	.7063162427192686
35	.8143832388747152
40	.891231809817949
45	.9409758994657749
50	.9703735795779884
55	.9862622888164461
60	.994122660865348
65	.9976831073124921
70	.9991595759651571

The rule of 72

If you want to know how long it will take you to double your money compounded regularly at x%, all you need to do is to divide 72 by x.

For example, and 4% interest compounded regularly your money will double in $\frac{72}{4} = 18$ years.

A Famous Numerical Loop

- You are asked to follow two rules as you select *any* natural number:
- **If the number is *odd* then multiply by 3 and add 1.**
- **If the number is *even* then divide by 2.**

- Regardless of the number you select, after continued repetition of the process, you *will always* end up with the number 1.

Let’s try it for the *arbitrarily selected* number 7.

7 is odd, therefore, multiply by 3 and add 1 to get: $7 \cdot 3 + 1 =$ **22**
22 is even, so we simply divide by 2 to get **11**
11 is odd, so we multiply by 3 and add 1 to get **34**.
34 is even, so we divide by 2 to get **17**.
17 is odd, so we multiply by 3 and add 1 to get **52**.
52 is even, so we divide by 2 to get **26**.
26 is even, so we divide by 2 to get **13**.
13 is odd, so we multiply by 3 and add 1 to get **40**.
40 is even, so we divide by 2 to get **20**.
20 is even, therefore, divide by 2 to get **10**.
10 is even, therefore, divide by 2 to get **5**.
5 is odd, so we multiply by 3 and add 1 to get **16**.
16 is even, so we divide by 2 to get **8**.
8 is even, so we divide by 2 to get **4**.
4 is even, so we divide by 2 to get **2**.
2 is also even, so we again divide by 2 to get **1**.

A word of caution!

- When you begin with 9,
  - it will take 19 steps

- When you begin with 41,
  - it will take 109 Steps

Getting into an Endless Loop

- Choose a 4-digit number (not one with all four digits the same).
- Rearrange the digits to make the biggest and smallest number.
- Subtract the two numbers.
- With this new number, continue this process.
- Soon you ***will*** get 6,174.
- But keep going!
- What do you notice?

We will (randomly) select the number 3,203

- The largest number formed with these digits is: 3320.
- The smallest number formed with these digits is: 0233.
- The difference is: 3087.
- The largest number formed with these digits is: 8730.
- The smallest number formed with these digits is: 0378.
- The difference is: 8352.
- The largest number formed with these digits is: 8532.
- The smallest number formed with these digits is: 2358.
- The difference is: 6174.
- The largest number formed with these digits is: 7641.
- The smallest number formed with these digits is: 1467.
- The difference is: 6174.

- And so the loop is formed, since you keep on getting 6174 if you continue

Another loop!

- Get the sum of *the squares* of the digits of any number:

- Let us use the number 5.
- Keep on taking the sum of the squares of the digits.

$5^2 = 25$, $2^2 + 5^2 = 29$, $2^2 + 9^2 = 85$, $8^2 + 5^2 = \mathbf{89}$,

$8^2 + 9^2 = 145$, $1^2 + 4^2 + 5^2 = 42$, $4^2 + 2^2 = 20$, $2^2 + 0^2 = 4$, $4^2 = 16$,

$1^2 + 6^2 = 37$, $3^2 + 7^2 = 58$, $5^2 + 8^2 = \mathbf{89}$, **...**

Mathematics can be entertaining.

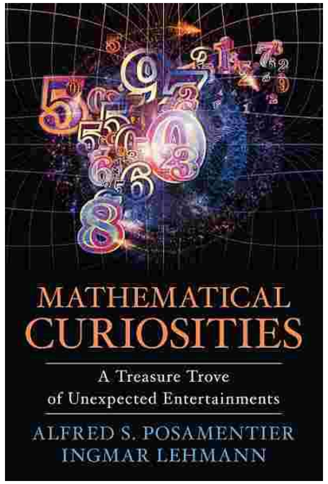
- In mathematics, the peculiarities are ***amazing!***

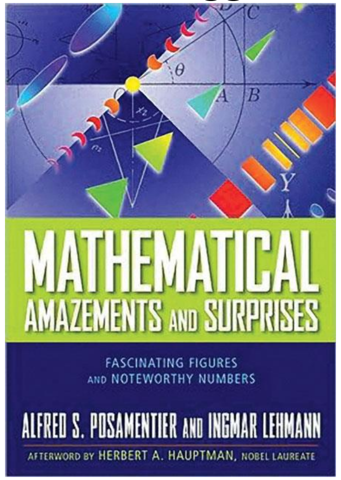
- Most concepts are interconnected; and the fun is to discover these connections.

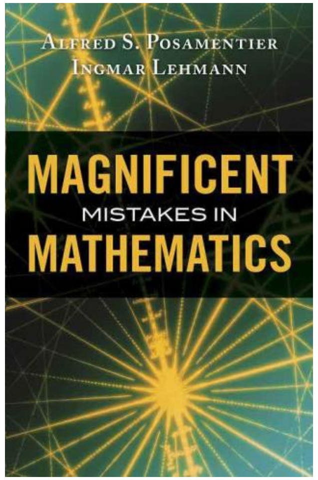
- There are endlessly many
  - ***amazements and surprises***

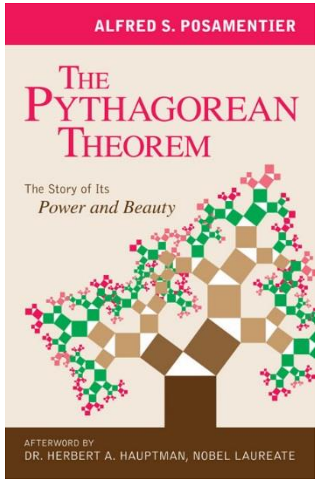
awaiting you!

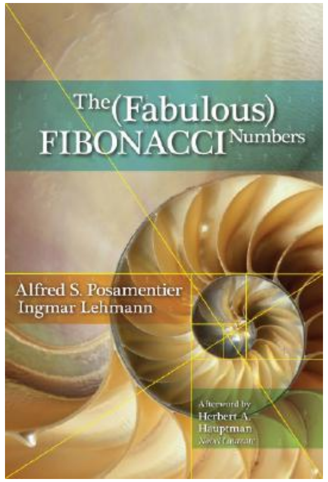
Some suggested sources from: www.Prometheus Books.com

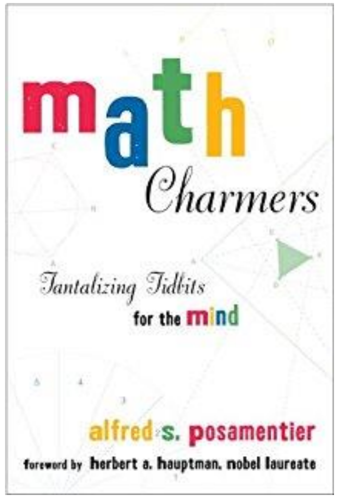


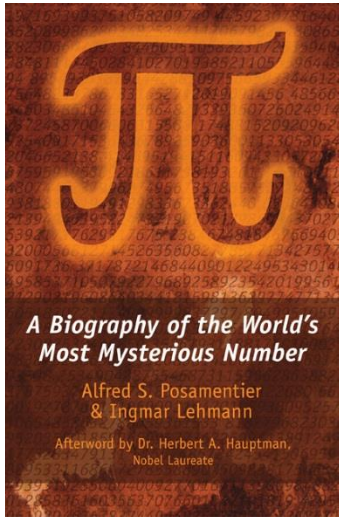


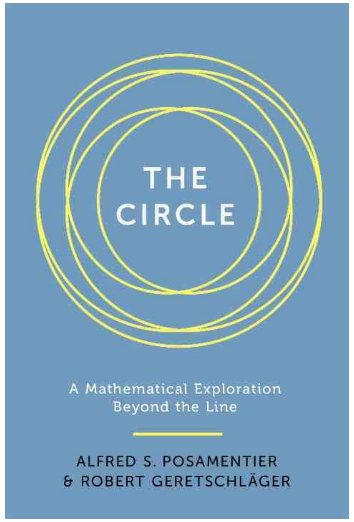


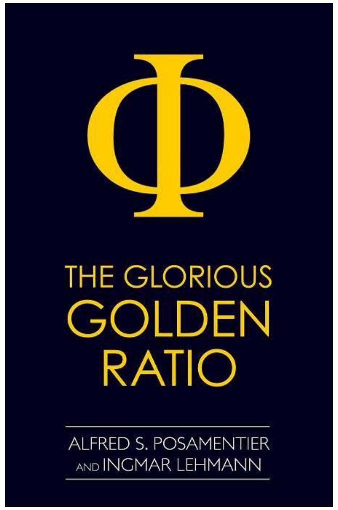


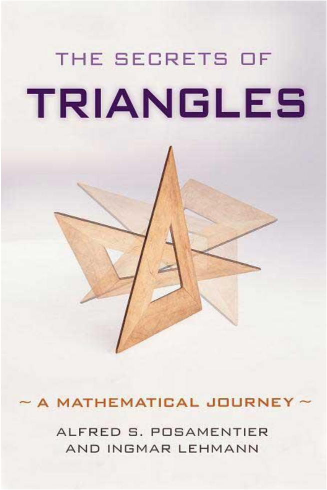


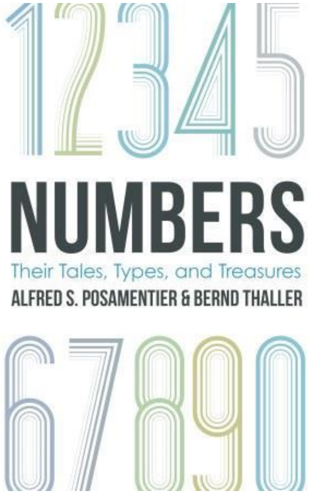


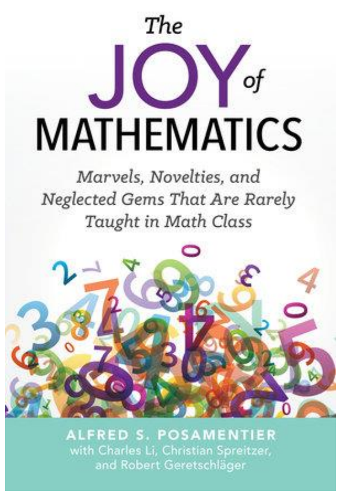












I hope you enjoyed the entertaining aspects of mathematics!

- Any questions?
  - *Please ask now!*

- **Or contact me at:**

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